Explosive free-electron-laser instability in a plasma-filled waveguide

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A relativistic electron beam passing through a plasma excites the microwave with phase velocity $v_{ph} < v_0$, where v_0 is the velocity of the relativistic electrons. At moderately high amplitude this microwave couples to a fast free-electron-laser mode (with $v_{ph} > c$) via a negative-energy beam mode, and this process leads to the explosive growth of all the three waves at the expense of the energy of the beam. The explosion time can be decreased as long as reasonable parameters are chosen.

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I. INTRODUCTION

The electromagnetic wiggler has been employed for generating free-electron-laser (FEL) radiation with much shorter wavelength [1–6]. This wiggler is usually an electromagnetic wave propagating opposite to the relativistic electron beam (REB). However, a few years ago, Tripathi and Liu [7] proposed an attractive scheme in which the free-electron-laser instability could have an explosive growth. They consider a slow electromagnetic wave wiggler and a REB propagating in the same direction in a medium. In this case, the wiggler and the REB couple to a fast electromagnetic wave via a negative-energy beam mode, and all the three waves grow simultaneously. Later, Sharma and Tripathi [8] investigated a more realistic situation in which the boundary conditions are included for the explosive FEL instability. In Ref. [9], they also suggest a Whistler wiggler FEL in which a plasma medium is employed. Nevertheless, they have not considered the effect of the boundaries in this problem.

In this paper we study the explosive FEL instability with a slow microwave wiggler propagating parallel to the REB in a plasma-filled cylinder, so the effect of the boundaries is included. In addition, the slow microwave wiggler is initially excited via the beam-plasma interaction. We also use an infinitely strong guide magnetic field to constrain the particle motion to be along the axial direction. However, both the beam and the plasma are assumed to be cold, and ions in the plasma are treated as motionless. It may be mentioned that the problem studied here is different from one studied by Lalita and Tripathi [10]. In Ref. [10], Lalita and Tripathi consider a Langmuir wave propagating opposite to the REB to be the pump in a cylinder filled with a warm plasma; thus, the interaction only gives rise to the nonexplosive instability. They also do not employ the guide magnetic field in their problem.

In Sec. II we study the linear instability caused by the excitation of the slow microwave via the beam-plasma interaction. In Sec. III we study the explosive growth of the instability, and in Sec. IV a brief discussion is given.

II. EXCITATION OF THE SLOW MICROWAVE

Consider a cylindrical waveguide of radius R filled with a uniform cold plasma of density n_0. A solid REB of radius r_b injects into it with velocity $\mathbf{v}_0 = v_0\hat{\mathbf{z}}$ and density n_{b0}, as shown in Fig. 1. As De Groot *et al.* recently proposed [11], this system may excite a very high-power slow microwave via the beam-plasma interaction:

$$E_z = E_0(r)\exp[i(kz-\omega t)] , \tag{1}$$

where ω and k are the angular frequency and wave number. Because we have employed an infinitely strong guide magnetic field, the particle motion is only along the z axis, and only the TM mode can be excited. However, ions in the plasma are assumed to be motionless.

The linear responses of plasma electrons and REB electrons to this microwave can be obtained by solving the equations of continuity and motion. The results are as follows:

$$n_p = \frac{en_0kE_z}{im\omega^2} , \tag{2}$$

$$v_p = \frac{eE_z}{im\omega}\hat{\mathbf{z}} , \tag{3}$$

$$n_b = \frac{en_{b0}kE_z}{im\gamma_0^3(\omega-kv_0)^2} , \tag{4}$$

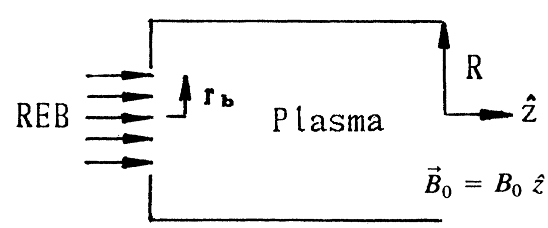

FIG. 1. Geometry of device. A solid REB passes through a cylindrical waveguide filled with a plasma. The system is immersed in an infinitely strong guide magnetic field ($B_0 = \infty$).

$$\boldsymbol{v}_b = \frac{eE_z}{im\gamma_0^3(\omega - kv_0)}\hat{\mathbf{z}} , \quad (5)$$

where n_p and $\boldsymbol{v}_p$ denote the perturbed electron density and velocity of the plasma, n_b and $\boldsymbol{v}_b$ describe the perturbed electron density and velocity of the REB, and $-e$ and m are the electron charge and rest mass. The $\gamma_0=(1-v_0^2/c^2)^{-1/2}$ is the relativistic factor.

Substituting (2), (3), (4), and (5) into the Maxwell equations, we obtain

$$\nabla_\perp^2 E_z + \left[1 - \frac{\omega_{pe}^2}{\omega^2} - \frac{\omega_{pb}^2}{\gamma_0^3(\omega - kv_0)^2}\right]\left[\frac{\omega^2}{c^2} - k^2\right]E_z = 0 , \quad (6)$$

where $\omega_{pe}^2=(4\pi e^2 n_0)/m$, $\omega_{pb}^2=(4\pi e^2 n_{b0})/m$, and

$$\nabla_\perp^2 = \frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial r}\right] + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} .$$

Specially, we assume that $r_b=R$. With the boundary condition $E_z(r=R)=0$, the dispersion relation can be obtained by solving Eq. (6). The result is

$$\left[1 - \frac{\omega_{pe}^2}{\omega^2} - \frac{\omega_{pb}^2}{\gamma_0^3(\omega - kv_0)^2}\right]\left[\frac{\omega^2}{c^2} - k^2\right] = \alpha_n^2 , \quad (7)$$

where α_n is the nth root of the equation $J_0(\alpha_n R)=0$, and J_0 is the zero-order Bessel function.

It is difficult to solve Eq. (7) analytically without any approximation, so we consider a special case, viz., $\omega_{pe}^2=\omega_{pb}^2/\gamma_0^3=\omega_p^2$. Then Eq. (7) can be solved for the slow microwave with $\omega^2/c^2<k^2$. The maximum growth occurs when

$$k^2 v_0^2 = \frac{3\omega_p^2}{\Delta} , \quad (8)$$

where $\Delta = 1+\alpha_n^2/k^2$. The result is

$$\omega = \tfrac{1}{2}kv_0 + \frac{1}{2}\frac{\omega_p}{\sqrt{\Delta}}i = \frac{\sqrt{3}\omega_p}{2\sqrt{\Delta}} + \frac{1}{2}\frac{\omega_p}{\sqrt{\Delta}}i , \quad (9)$$

giving the maximum growth rate

$$\Gamma_{\max} = \mathrm{Im}(\omega) = \frac{\omega_p}{2\sqrt{\Delta}} . \quad (10)$$

The real part of ω is written as

$$\omega_r = \mathrm{Re}(\omega) = \frac{kv_0}{2} = \frac{\sqrt{3}\omega_p}{2\sqrt{\Delta}} , \quad (11)$$

or

$$\omega_r = kv_0 - \omega_r . \quad (12)$$

The slow microwave thus generated has a phase velocity $\boldsymbol{v}_{\rm ph}=w_r/k=v_0/2<v_0<c$. This wave, traveling parallel to the direction of the beam, is likely to couple to a negative-energy beam mode and a high-frequency FEL wave mode in such a way that all the three waves grow explosively. This process will be investigated in detail in Sec. III.

III. EXPLOSIVE INSTABILITY

Consider a forward-propagating slow microwave mode (ω_i,k_i) coupling to a negative-energy beam mode $(\omega_l k_l)$ and a high-frequency FEL wave mode (ω_s,k_s). The phase-matching conditions $\omega_l=\omega_s+\omega_i$ and $k_i=k_s+k_i$ lead to the following expression for the FEL frequency:

$$\omega_s = 2\gamma_0^2(k_i v_0 - \omega_i) . \quad (13)$$

If the slow microwave (ω_i,k_i) is excited via the mechanism studied in Sec. II, Eq. (13) can be rewritten as

$$\omega_s \approx 2\gamma_0^2\omega_i , \quad (14)$$

according to Eq. (12).

We assume that $\omega_s \gg \omega_{pe}$, so that the plasma electrons do not play an important role during the interaction process [10]. Following the method used by Sharma and Tripathi in Ref. [8], one may derive the nonlinear responses of REB electrons to the three waves from the equations of continuity and motion. The results are as follows:

$$\begin{aligned} \mathbf{J}_i^{\rm NL} &= -A\omega_i E_{zl}E_{zs}^*\hat{\mathbf{z}} , \\ \mathbf{J}_s^{\rm NL} &= -A\omega_s E_{zl}E_{zi}^*\hat{\mathbf{z}} , \\ \mathbf{J}_l^{\rm NL} &= A\omega_l E_{zs}E_{zi}\hat{\mathbf{z}} , \end{aligned} \quad (15)$$

where $\mathbf{J}_i^{\rm NL}$, $\mathbf{J}_s^{\rm NL}$, and $\mathbf{J}_l^{\rm NL}$ represent the nonlinear current densities driving the slow microwave mode (ω_i,k_i), the high-frequency FEL wave mode (ω_s,k_s), and the beam mode (ω_l,k_l), respectively, and

$$A = \frac{e\omega_{pb}}{4\pi m\gamma_0^{9/2}(\omega_i - k_i v_0)(\omega_s - k_s v_0)} \times \left[\frac{k_s}{\omega_s - k_s v_0} + \frac{k_i}{\omega_i - k_i v_0} + \frac{k_i\gamma_0^{3/2}}{\omega_{pb}} - \frac{3v_0\gamma_0^2}{c^2}\right] . \quad (16)$$

Also, E_{zi}, E_{zs}, and E_{zl} are the longitudinal electric fields of the wave modes (ω_i,k_i), (ω_s,k_s), and (ω_l,k_l), respectively. They can be written as

$$E_{z\alpha} = E_\alpha(t)J_0(\alpha_n r)\exp[i(k_\alpha z - \omega_\alpha t)] , \quad (17)$$

where $\alpha=i,s,l$. In writing Eq. (17), we have used the assumption of $r_b=R$.

We employ (15) and (17) in the Maxwell equations, and we assume that the mode structures of E_{zi}, E_{zs}, and E_{zl} remain unmodified (only the eigenvalues are changed) and that only the single modes of three waves interact resonantly (see Ref. [8]). Then we get

$$\begin{aligned} \frac{\partial E_i}{\partial t} &= 2\pi\omega_i AgE_l E_s^* , \\ \frac{\partial E_s}{\partial t} &= 2\pi\frac{\alpha_n^2 c^2}{\omega_s}AgE_l E_i^* , \\ \frac{\partial E_l}{\partial t} &= 2\pi\frac{\omega_{pb}}{\gamma_0^{3/2}}AgE_i E_s , \end{aligned} \quad (18)$$

where

$$g=\frac{2}{\alpha_n^2 R^2 J_1^2(\alpha_n R)}\int_0^{\alpha_n R} J_0^3(x)x\,dx \; . \tag{19}$$

Defining the action amplitudes a_i, a_s, a_l as

$$E_i=a_i e^{i\phi_i} \; ,$$
$$E_s=\frac{\alpha_n c}{\sqrt{\omega_i\omega_s}}a_s e^{i\phi_s} \; ,$$
$$E_l=\sqrt{\omega_{pb}/\gamma_0^{3/2}\omega_i}\,a_l e^{i\phi_l} \; ,$$

we obtain the following set of equations:

$$\frac{\partial a_i}{\partial t}=Pa_l a_s\cos\Phi \; ,$$
$$\frac{\partial a_s}{\partial t}=Pa_l a_i\cos\Phi \; , \tag{20}$$
$$\frac{\partial a_l}{\partial t}=Pa_i a_s\cos\Phi \; ,$$

where $\Phi=\phi_l-\phi_i-\phi_s$, and

$$P=\left[\frac{4\pi^2\alpha_n^2 c^2 A^2 g^2\omega_{pb}}{\omega_s\gamma_0^{3/2}}\right]^{1/2} . \tag{21}$$

Following Liu and Tripathi [12], the explosion time for the case $a_i(0)>a_s(0)$, $a_l(0)=0$ turns out to be

$$\tau_\infty=\frac{1}{Pa_i(0)}=\frac{\gamma_0^{3/4}}{2\pi\alpha_n c a_i(0)}\left[\frac{\omega_s}{\omega_{pb}A^2g^2}\right]^{1/2} . \tag{22}$$

The saturation of the explosive instability may occur when the particles are trapped in the beam mode. However, only through the computer simulations can the level of saturation be determined.

IV. DISCUSSION

The explosion time is sensitive to the amplitude $a_i(0)$. Stronger coupling occurs when $a_i(0)$ becomes larger, so the explosive instability plays an important role in this system at a moderately high amplitude of the slow microwave excited via the beam-plasma interaction. However, the slow microwave can be launched from an external source and can become an efficient wiggler for the generation of the high-frequency FEL. The boundary condition is significant for decreasing the explosion time through the parameters α_n and g. Other parameters should also be carefully chosen for giving a smaller explosion time. Typically, we choose the following parameters, according to Ref. [11]: $r_b=R\approx 3$ cm, $\gamma_0\approx 2$, $\omega_{pe}\approx\omega_{pb}/\gamma_0^{3/2}\approx 50$ GHz (thus $\omega_i\approx 43$ GHz), $ea_i(0)/mc\approx 5$ GHz. We obtain $\omega_s\approx 344$ GHz. The explosion time turns out to be a few nanoseconds for the TM_{01} mode.

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